Academy At Home

# Pre Grade 10 Assessment Test for Mathematics

S means student

Always watch for careless errors in every question _ B/C

**Solutions in yellow box**

Try to keep the student **relaxed** – **talking aloud** all the time and using **pen and paper**. Give **minimal hints** if appropriate but do not take on the role of tutor – **you are diagnosing – not fixing problems!**

**Key to Shorthand**

| | | |
|---|---|---|
| T/A _ Talking Aloud | Und _ Desire for Understanding | D _ Defensive |
| B/C _ Backchecking | L/T _ Lazy Thinking | OK/W _ OK to be wrong |
| P/P _ Pen & Paper | P/S/T _ Positive Self Talk | W/R _ Willing to take risks |
| S/Q _ Self questioning | R/D _ Rule Dependency & patternizing | Lat/T _ Lateral Thinking |
| M/P/T _ Memory Pegs and Triggers | P/S _ Problem Solving Skills | M/L _ Math Language is weak |

Space is left between questions so you can **make notes AS you go along!**

1) Find the value of $\frac{2}{3} \div \frac{5}{9} \times \frac{5}{2}$

$$\frac{2}{3} \times \frac{9}{5} \times \frac{5}{2} = \frac{1}{1} \times \frac{3}{1} \times \frac{1}{1} = 3$$

**Gap** – if S can't handle knowledge about how to **M/T** [better memory triggers needed] – only the sight of a "+" or " -" with fractions should trigger the concept 'common denominator'.

If doesn't divide out – ask S to do multiply, show S how to divide out an d **see if S cares!**

Point out again that **all algebra is based on arithmetic and see if S cares!**

*Note to assessor: should divide out wherever possible*

2) Evaluate $2 + 3 \times 5$

**2 + 15 = 17**

If S says "5 × 5 =25" and then doesn't recognize BEDMAS or order of operations when you give minimal hints, BEDMAS is a **Gap**!

If S says "5 × 5 =25" and then does know BEDMAS when questioned, S must begin learning **M/T** – in this case, the sight of various operations [+ and × ] must trigger BEDMAS.

If jumps to an incorrect answer such as 25 - possibly L/T and/or P/P

3) Find the value of $(2+3-1)^2$

**$= 4^2 = 16$**

If S begins to distribute – definitely is **R/D**!
If does 4+9+1 or 4+ 9-1 – do the following example and you can judge if S really wants to **understand**!!! ↓
$(2+3)^2$ Do this two ways: 1) 4+9=13 and $(5)^2$ =25 and ask which one S **knows** is right. S will know it is 25 – point out that all algebra is based on arithmetic. Judge if S cares or not – important that S **wants** understanding!!!!

4) What is the value of a) $\frac{0}{2}$ b) $\frac{2}{0}$ c) $\frac{0}{0}$

a) What do you multiply 2 by to get 0 _ answer **0**
b) What do you multiply 0 by to get 2 _ answer **impossible**
c) What do you multiply 0 by to get 0 _ answer **any number**

S will likely guess at some of these; S might say "0" for all three.
Explain that you are going to teach S how to teach him/her self – rather than rely on rules or memory. **R/D**
Write $\frac{12}{4}$ and tell S that he/she is teaching a Grade 3 class – all of whom are excellent multipliers, but have never heard the word division. Ask S to think of a question using the word multiply to which the Grade 3 class would immediately give the correct answer of 3!!
Assess S's **trust and self confidence and ability** to come up with "what do you multiply 4 by to get 12?"
You may have to give gentle hints – but avoid telling S the question.
Ask S to apply this same question to parts a, b, c.

a) should be easy
b) Assess S's willingness to risk saying impossible – **W/R** – do not accept undefined.
c) Assess S's **logic** and **Lat/T** to come up the answer "any number".

5) Express 64 as a power of 2

**$= 2^6$**

Explain base is two right away – if S still cannot begin – probably **M/L** a difficulty. **Help get S started!**
If does in head well – likely **verbal**. If does on paper well – likely **visual**. If does in head incorrectly, likely needs **P/P**
Watch for the need to **B/C**
**Gap** if S can't handle exponents and powers.

6) Find the value of $2+\frac{1}{3}+\frac{3}{7}$

$$2+\frac{1}{3}+\frac{3}{7}=\frac{42}{21}+\frac{7}{21}+\frac{9}{21}=\frac{58}{21}$$

**Gap** –if can't apply a common denominator.

Watch that S knows how to handle the 2 as $\frac{42}{21}$

If S has trouble and says "haven't done this type of question for ages" – S is **defensive** and this can be serious trouble at this level. Be "cool" and explain again that Algebra is based on Arithmetic and these are the reasons S is having trouble in Algebra

7) How many quarter kilos are there in $5\frac{1}{2}$ kilos?

4 quarters in one whole so 4 times 5 is 20 and then two more quarters in one-half so 20 plus 2 is 22.

If S seems unsettled get S to explain **why** and may find a fear of fractions or a fear of the word "kilos" – needs **OK/W** strategy.

If S begins to manipulate the fractions randomly [such as $5\frac{1}{2}\times\frac{1}{4}$ or even $5\frac{1}{2}\times 4$ [which leads to the correct answer] – ask **why** and S will likely admit to wishful thinking. **R/D**

Encourage S to use his/her own logic – and if S is still unsure – suggest he substitute a friendlier word in place of "kilo"– **Lat/T** – such as "pizza".

If and when S suddenly sees that all that has to be done is 5 times 4 [since 4 quarters in one whole] and then add 2 for the extra half – commend S for good logic and suggest that S should rely on his or her **logic first** – before falling into patternizing blindly.

8) For a new motor bike, the recommended ratio of gas to oil is 25:1, by volume. How much gas and how much oil are needed to fill a 13 L tank on a new motor bike?

Several different approaches [here are 2]:

Let oil be 25x and gas be 1x _ 26x = 13

$\therefore$ x = 0.5 and $\therefore$ oil is 12.5L and gas is 0.5L

or

Let yL be am't of oil $\therefore \frac{25}{26}=\frac{y}{13}$ – $\frac{25}{2}=\frac{y}{1}$

$\therefore$ 2y=25 and 12.5L of oil and 0.5L of gas.

**R/D** and/or **L/T** if slaps down 'equal' ratios without thought such as $\frac{25}{1}=\frac{13}{?}$.

**Anxiety** over **ratios** and/or **word problems** if can't start, wants to cooperate, and appears anxious

**Global problem solver** if talks a lot – much of it correct [such as 25+1 is 26 and this corresponds to 13L], but cannot get focused and really begin.

**Microscopic** if deals well with one part and then can't see beyond it [such as writing $\frac{25}{1}=\frac{\text{oil}}{\text{gas}}$ and puzzles over this – not seeing the use of 13]

**Gap** – if the concept of ratios is still foreign after being given some hints.

9) Simplify $5 - 2(2x - 3)$

$= 5 - 4x + 6$
$= 11 - 4x$

If S subtracts first, writing $3(2x-3)$ it is most likely due to one of the following and by questioning, you should determine which one.

? **Microscopic problem solver** [jumps in as soon as sees something familiar – doesn't back away and plan]

? **Lack of memory triggers** – the sight of both "-" and "x" should trigger BEDMAS [order of operations] and you will know from earlier work if S knows BEDMAS.

? **Gap** in order of operations – least likely by Grade 11

If S writes $= 5 - 4x - 6$, there may be a mild **perceptual** problem [the '-' sign is mentally carried over] or S may just be **'jumping'**

10) Solve for x: $9 - 7x = 11 - 5x$

$-7x+5x=11 - 9$
$-2x = 2$
$x = -1$

Watch for **genuine understanding** when S manipulates the terms. Ask S '**why**' he/she wrote +5x on the left – if S says something like "I took it over and changed the sign" – pretend S is teaching you and say "what is take over? – all I know how to do is $+,-,\times,\div$ - if S insists it is **just a rule** **[R/D]** – give S $2x=10$ and tell S to "take over" the 2 – see if S incorrectly changes the sign.
Explain that to get rid of the "- 5x" on the right side, one must **add** "5x" **to both sides**. **Judge whether S cares at all about understanding!**
Watch for "careless" errors that **B/C** would eliminate.

11) If $a = -3$ find the value of $-a^2$

$-a^2 = -(\ )^2 = -(-3)^2 = -(9) = -9.$

If S does it all in his/her head and gives the wrong answer of +9 or even the right answer of -9, ask S to explain how he/she got that answer – likely needs to learn **P/P**
Often S jumps to the rule "a – times a – is +" so the answer is +9. **R/D** and **L/T**
Do not correct – say "pretend you're from Mars, and Martians have the same symbols for "-", "2", but instead of the letter "a" they write "( )" Tell S to rewrite $-a^2$ in Martian – S should write $-(\ )^2$ - then tell S to substitute the value of "a" in the empty bracket. **Gap** in substitution.
If S still has difficulty, it will show a **Gap** in Order of Operations.

12) If 9% of a number is 72, find the number.

$\frac{9}{100} \times x = 72$

$9x = 7200$

$x = 800$

The number is 800

If S has difficulty expressing '9% of an unknown number is 72', then S needs work on the skill of: **Translating English into Math.**
**Anxiety** may express itself with % and fractions and if that is the case you will notice avoidance or jumping in without thought or just giving up.
A **Gap in %** may exist, but by Grade 10, more apt to be one of the above.

13) Factor $9a^3b^2 - 12a^2b^4 + 3a^2b^2$

$=3a^2b^2(3a - 4b^2+1)$

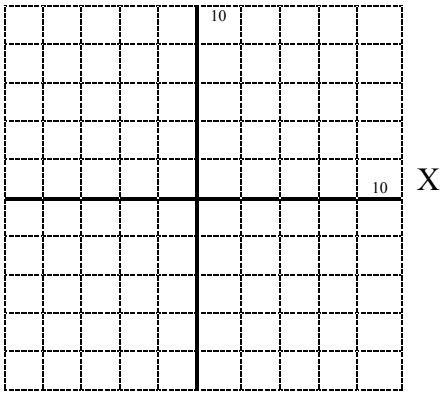
**Gaps** in exponential work and common factoring will show up in this question.
Note: Assess whether S knows the 'golden rule' of factoring: **always common factor first** – if not a **Gap**. **B/C** will point out the last term in the bracket as "+1", if missed when first doing the question.
If S has difficulty and you help S find the common factor or of $3a^2b^2$– get S to ask himself/herself the generic questions "what do I multiply $3a^2b^2$ by to get $9a^3b^2$" and so on. S must develop good **self-questioning skills.**

14) Sketch the graph of $3x - y - 2 = 0$

There are two ways S might use.
- Isolate y and get $y = 3x - 2$

Mark the y intercept (0,-2) on the graph, and then find another point using the fact that the slope is 3 so from (0,-2) move one to the right and three up – may continue finding more points in the same way. Then draw a straight line
- Use a table of values and plot these points and draw a straight line

| x | y |
|---|---|
| 0 | -2 |
| 2 | 4 |
| 4 | 10 |




Does S recognize this is a straight line? If not – needs **M/T** – a linear equation in x and y [linear _ $x^1$ and $y^1$, never any other exponents] always means a straight line! This **M/T** should prompt the **M/P** that S uses to graph a straight line. The **M/P** tells S to set up a **table of values** [or **isolate y** and use the other approach with **slope**]
**Gap** – if S has no idea what to do even with many hints.

15) Evaluate $\frac{2^{12}}{2^4}$, $2^0$, and $2^{-1}$

$\frac{2^{12}}{2^4} = 2^8$ [may leave it as this]

$2^0 = 1$

$2^{-1} =$ _

If, with $\frac{2^{12}}{2^4}$, S says $2^3$, ask S to teach you "why" this is so. If S again says, "divide 4 into 12", tell S to write out what $2^{12}$ and $2^4$ means and with $\frac{2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2}{2 \bullet 2 \bullet 2 \bullet 2}$, tell S to divide out.
When S arrives at the correct answer of $2^8$, ask S to state the "rule" for dividing powers with the same base. If S says –"I knew that – I just forgot " – be careful about S being **'defensive'** – tell him that knowledge in his head must be accessible – he needs **M/T** – in this case the sight of a power divided by a power must instantly access the **M/P** [rule] just stated. Check that he **understands** the words 'base' and 'power' **[M/L].** Ask if he understands 'why' this is the rule – assess whether S cares **about understanding** or not.
Then go to $2^0$, and whether S gives the correct answer of 1 or not, have S teach **'why'** he arrived at his answer. Help S by looking at $\frac{2^3}{2^3}$ - S will know that this is "**1**" – then have him apply the rule above and he will arrive at $2^0$=1. Similarly, $\frac{2^2}{2^3} = \frac{4}{8} = \frac{1}{2}$ and the rule stated above tells S that $\frac{2^2}{2^3} = 2^{-1} = \frac{1}{2}$ and work out the 'rule' here.
Does S care about understanding why a rule works, or does S still just want a rule without thought? **Wanting to understand and not just follow rules blindly, is a key component for success!**

16) Solve $\frac{x}{3} - \frac{3x}{2} = \frac{1}{6} - x$

Multiply both sides by 6

$6 \bullet \frac{x}{3} - 6 \bullet \frac{3x}{2} = 6 \bullet \frac{1}{6} - 6 \bullet x$

Divide out correctly and arrive at
$2x - 9x = 1 - 6x$
$2x - 9x + 6x = 1$
$- x = 1$
$x = - 1$

If S keeps a common denominator of 6 [rather than multiplying through by 6 and arriving at an equation with no fractions], do not correct him at this point. Make a note about **efficiency**.
Sometimes, S will multiply only one side or multiply the sides by different numbers – watch for a real **understanding** about the meaning of an **equation** and **balancing both sides**.
Watch for **B/C**.
When finished, if necessary, show S how to get rid of fractions immediately [multiply every term on each side by 6] and briefly discuss **efficiency** – is S **short of time** on tests?

17) Determine the equation of a line perpendicular to $x - 5y + 2 = 0$ and passing through the point (1,2)

Find the slope of $x - 5y + 2 = 0$ [isolate y]
$-5y = -x - 2$
$y = \frac{1}{5}x + \frac{2}{5}$ $\therefore$ slope is $\frac{1}{5}$ and the slope of the perpendicular line is $-5$
Now we want the equation of a line with $m = -5$ and passing through (1,2)
Use either of the following two formulae.

I $y - y_1 = m(x - x_1)$ $\therefore$ $y-2 = -5(x-1)$ _ $y - 2 = -5x + 5$
$\therefore$ equation of new line is $5x + y = 7$

OR

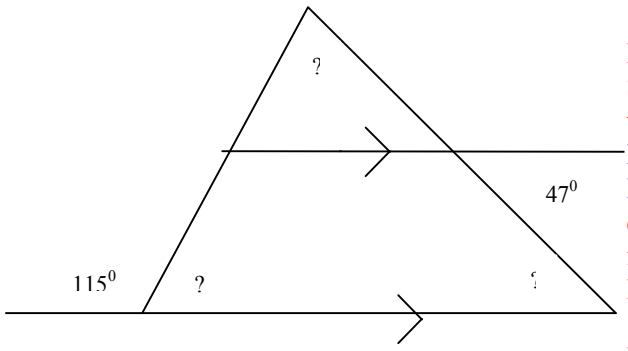
II $y = mx + b$ _ $y = -5x + b$
Subst. in point (1,2) _ $2 = -5(1)+b$ _ $2 = -5 + b$ $\therefore$ $b = 7$
$\therefore$ equation of new line is $y = -5x + 7$

This question will show how well S is able to handle concepts in linear geometry.
Does S know how to find the slope of a given line?
- If S can continue with just a brief hint, then **M/T** is the problem – S has the concepts in his/her head but cannot access them when needed!
- If S shows no knowledge of the above even with very leading hints such as 'isolate y, now where is the slope?' then the above concept is a **Gap.**

Does S know how to find a perpendicular slope?
- If not, likely a **Gap**.

Does S know how to find the equation of a line, given the slope and a point?
- If S can access one of the two methods shown to the left with just a brief hint, then **M/T** is the problem – S has the concepts in his/her head but cannot access them when needed!
- If S shows no knowledge of the above even with very leading hints such as 'do you remember any formula used for a straight line? It begins with $y - y_1$, if that draws a blank try $y = m\ldots$' and if still no memory, then the above concept is a **Gap**.

18) Find the value of each angle marked with a ?

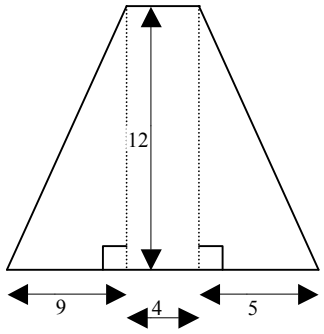



Left-hand ? = $65^0$ [$180^0$ in a straight line]
Right side ? = $47^0$ [Parallel lines and Z angles]
Top ? = $68^0$ [angles in a triangle add up to $180^0$]

If S balks with this question, it may show:
- **anxiety** with Geometry
- an **unwillingness to try** if not sure how to answer **all** of the question

If S begins with the top ? and keeps struggling with it – S does not know how to back away and look at the whole question in order to **plan**. S will need to learn the **Problem Solving Question, "what am I given and how can I use it"** – this will focus S's attention on either the $115^0$ angle or the $47^0$ angle and thus help S find one of the '?' angles.
If necessary, tell S to look at the $115^0$ angle, and find the '?' beside it. If S still does not know – **Gap**.
If necessary, tell S to look at the $47^0$ angle, and find the '?' at the bottom right. If S still does not know – **Gap**.
When S has found the two bottom '?', and cannot find the top '?', ask S to back away and look at the whole question – ask S what he/she sees – if necessary tell S that he/she is looking at a triangle. If S still cannot find the top '?' – **Gap**.

19) Find the perimeter and the area of the figure shown below.




Use Pythagoras to find the left hand sloping side.
$x^2 = 12^2 + 9^2$. $\therefore$ $x^2 = 144 + 81$ _ $x^2 = 225$ _ $x = 15$
Use Pythagoras to find the right hand sloping side.
$y^2 = 12^2 + 5^2$. $\therefore$ $y^2 = 144 + 25$ _ $y^2 = 169$ _ $y = 13$
Perimeter = 15+9+4+5+13+4 = 50
Two ways to find the area:

1) Add up the areas of the two triangles $[\frac{1}{2} \bullet b \bullet h]$ and the rectangle $[b \bullet h]$ in the middle .

Area = $\frac{1}{2} \bullet 9 \bullet 12 + \frac{1}{2} \bullet 5 \bullet 12 + 4 \bullet 12$
= 54 + 30 + 48 = 132

2) Use the formula for a trapezoid

$A = \frac{1}{2} \bullet (a + b) \bullet h$ $\therefore$ $A = \frac{1}{2} \bullet (4 + 18) \bullet 12$

$A = \frac{1}{2} \bullet 22 \bullet 12 = 132$

If S says "I can't do it" or "I have no idea", ask S what 'perimeter' means. Then tell S to get started on the Perimeter. As S starts to work it out – tell S – "see you CAN begin and this will get you part marks if nothing else". This identifies **L/T**.
If S says " I haven't done stuff like this for years" - S is being **defensive** and this must be handled carefully. Explain that as S progresses to the higher Grades, he will need to use MUCH of the Math he has 'learned' before.
If S looks panicky and says something like "I hate this stuff", S is **anxious**.

In every case however, S will need good **Problem Solving Skills!**
**Q: "What am I given and how can I use it?"**
Possible answers:
- I see right angled triangles
- I see a bunch of measurements
- I see a trapezoid.

But S still may not be able to continue. S must learn **to jump back and forth** between the above question and the following one.
**Q: "What am I trying to find and what do I need to find it?"**
Now, S should focus in on the word 'perimeter' [if S tries to talk about P and A together, S is too **global** and needs to learn to tackle one idea at a time].
So S will answer:
"P = 9+4+5+4+ …."
With **T/A** S will say "I need to find the length of the sloping sides".
S should continue with "**how** do I find the left hand slide – oh it is the hypotenuse of a right angled triangle, so $x^2 = 12^2 + 9^2$, and then S should be fine!
If S doesn't recognize that he/she needs to use Pythagoras, but knows Pythagoras when questioned, then **M/T** is the problem [a right angled triangle should be the trigger for Pythagoras]. If S doesn't know anything about Pythagoras then it is a **Gap**.
A similar approach is needed to find Area.
If S says – "**I can't remember the formula** for the Area of a trapezium" and quits then S is **R/D**. S must learn that his first choice is his/her **own logic**!
Force S to find any area that he/she can in the figure and S will likely be able to find the total area – point out that **S must keep logic at the forefront ahead of 'rules'**